

VARIABLE RETURNS TO SCALE,
STABILITY AND TECHNICAL
PROGRESS WITH NONTRADED GOODS

Shigemi Yabuuchi*

1. Introduction

Nontraded goods was introduced into the standard model of international trade and some of the major trade theorems have been examined by Komiya (1967), followed by Kemp (1969), Ethier (1972), Ray (1972), Batra (1973), Flam (1979), Hazari, Sgro and Suh (1981), Kakimoto and Yabuuchi (1986) and so on. On the other hand, the implication of variable returns to scale (VRS) for the theory of international trade have been extensively studied by Jones (1968), Batra (1968), Herberg and Kemp (1969), Kemp and Negishi (1970), Mayer (1974), Eaton and Panagariya (1979), Panagariya (1980, 1981), Choi and Yu (1984, 1985) and so on. However, no attempt has been made to ascertain the role and impact of VRS on the trade theory in conjunction with nontraded goods except for recent contribution by Choi (1987). He explored the welfare implications of a change in the terms of trade and the rate of tariff, as well as the relative effectiveness of some trade policy instruments in the three-commodity (two traded and one nontraded) and two-factor trade model allowing the presence of VRS.

Herberg and Kemp (1969) has shown that under VRS the price-output

* The author is indebted to Professors Murray Kemp and Jai-Young Choi for many valued comments that led to considerable improvement on the earlier version of this paper.

response may be ambiguous. Mayer (1974) has shown that the output of a given commodity responds positively to an increase in its relative price in a dynamically stable system, and Neary (1978) has shown that a perverse price-output response cannot occur if stability in factor market is assumed. These analyses are confined to the model *without* nontraded goods. When Choi (1987) discussed price-output response, he has not considered dynamic stability for nontraded good case and just confined his analysis to the positive price-output response case.

The first purpose of this paper is to investigate the relation between the price-output response and dynamic stability with nontraded goods under VRS. It will be shown that even in a dynamically stable system some qualifications are required for the normal price-output response. Then the effects of (Hicks-neutral) technical progress in the nontraded good sector on outputs are also discussed, and Komiya's results on the effects of technical progress are extended to allow for VRS.

2. The model and assumptions

Let us assume an economy in which there are three industries producing two tradeable goods, X_1 (exportable) and X_2 (importable), and one nontraded good, X_3 . The economy is endowed with primary factors, labour L and capital K , their returns being denoted w and r respectively. In addition, it is assumed that returns to scale are variable and external to the firm. The production functions are :

$$X_j = g_j(X_j)F^j(L_j, K_j; t_j), \quad j=1, 2, 3, \quad (1)$$

where g_j describes the role of output-generated externality and is a positive function defined on $(0, \infty)$, L_j and K_j are labour and capital employed in the j th industry, respectively, and t_j is the parameter indicating its state of technology. F^j is homogeneous of degree one in the L_j and K_j and in t_j .

Under competition, we have :

$$a_{L1}w + a_{K1}r = 1 \quad (2)$$

VARIABLE RETURNS

$$a_{L2}w + a_{K2}r = p \quad (3)$$

$$a_{L3}w + a_{K3}r = q \quad (4)$$

where a_{ij} is the amount of the i th factor used in the j th industry to produce one unit of the output, p and q are the relative prices of the importable (second) and the nontraded (third) goods, respectively, in terms of the exportable (first) good. Exogenously given endowments impose resource constraints:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = L \quad (5)$$

$$a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 = K. \quad (6)$$

The model will be completed with the demand and supply equation for the nontraded good:

$$D_3(p, q, Y) = X_3, \quad (7)$$

where

$$Y = X_1 + pX_2 + qX_3. \quad (8)$$

Differentiating (2), (3) and (4), we get:

$$\theta_{L1}\hat{w} + \theta_{K1}\hat{r} = \hat{X}_1 - (\theta_{L1}\hat{L}_1 + \theta_{K1}\hat{K}_1) \quad (9)$$

$$\theta_{L2}\hat{w} + \theta_{K2}\hat{r} = \hat{X}_2 - (\theta_{L2}\hat{L}_2 + \theta_{K2}\hat{K}_2) + \hat{p} \quad (10)$$

$$\theta_{L3}\hat{w} + \theta_{K3}\hat{r} = \hat{X}_3 - (\theta_{L3}\hat{L}_3 + \theta_{K3}\hat{K}_3) + \hat{q} \quad (11)$$

where " $\hat{}$ " indicates relative changes, e.g., $\hat{w} = dw/w$ and θ_{ij} is the distributive share of the i th factor in the j th industry ($i = L, K$; $j = 1, 2, 3$), e.g., $\theta_{L2} = wa_{L2}/p$.

Differentiating (1), we obtain:

$$(1 - e_j)\hat{X}_j = \theta_{Lj}\hat{L}_j + \theta_{Kj}\hat{K}_j + T_j\hat{t}_j, \quad (12)$$

where $e_j (= F^j g_j')$ is the output elasticity of returns to scale of the j th industry and $T_j = (t_j/F^j)(\partial F^j/\partial t_j)$. Substituting (12) into (9), (10) and (11), we have:

$$\theta_{L1}\hat{w} + \theta_{K1}\hat{r} = e_1\hat{X}_1 + T_1\hat{t}_1 \quad (13)$$

$$\theta_{L2}\hat{w} + \theta_{K2}\hat{r} = e_2\hat{X}_2 + T_2\hat{t}_2 + \hat{p} \quad (14)$$

$$\theta_{L3}\hat{w} + \theta_{K3}\hat{r} = e_3\hat{X}_3 + T_3\hat{t}_3 + \hat{q}. \quad (15)$$

Let us define the elasticity of factor substitution as:

$$\sigma_j = (\hat{K}_j - \hat{L}_j)/(\hat{w} - \hat{r}) = (\hat{a}_{Kj} - \hat{a}_{Lj})/(\hat{w} - \hat{r}). \quad (16)$$

(12) and (16) yield:

$$\hat{L}_j = (1 - e_j) \hat{X}_j - T_j \bar{t}_j - \theta_{Kj} \sigma_j (\hat{w} - \hat{r}) \quad (17)$$

$$\hat{K}_j = (1 - e_j) \hat{X}_j - T_j \bar{t}_j + \theta_{Lj} \sigma_j (\hat{w} - \hat{r}). \quad (18)$$

Differentiating (5) and (6), we obtain :

$$\lambda_{L1} \hat{L}_1 + \lambda_{L2} \hat{L}_2 + \lambda_{L3} \hat{L}_3 = \hat{L} \quad (19)$$

$$\lambda_{K1} \hat{K}_1 + \lambda_{K2} \hat{K}_2 + \lambda_{K3} \hat{K}_3 = \hat{K}, \quad (20)$$

where λ_{ij} is the allocative share of the i th factor in the j th industry ($i = L, K$; $j = 1, 2, 3$), e.g., $\lambda_{L1} = L_1/L$. Substituting (17) and (18) into (19) and (20), we get :

$$\Lambda_{L1} \hat{X}_1 + \Lambda_{L2} \hat{X}_2 + \Lambda_{L3} \hat{X}_3 = \hat{L} + \sum_{j=1}^3 \lambda_{Lj} T_j \bar{t}_j + \delta_L (\hat{w} - \hat{r}) \quad (21)$$

$$\Lambda_{K1} \hat{X}_1 + \Lambda_{K2} \hat{X}_2 + \Lambda_{K3} \hat{X}_3 = \hat{K} + \sum_{j=1}^3 \lambda_{Kj} T_j \bar{t}_j - \delta_K (\hat{w} - \hat{r}) \quad (22)$$

where $\Lambda_{Lj} = (1 - e_j) \lambda_{Lj}$, $\Lambda_{Kj} = (1 - e_j) \lambda_{Kj}$, $\delta_L = \sum_{j=1}^3 \lambda_{Lj} \theta_{Kj} \sigma_j$ and $\delta_K = \sum_{j=1}^3 \lambda_{Kj} \theta_{Lj} \sigma_j$.

Differentiating (7) and (8), we derive :

$$\begin{aligned} & e_1 X_1 \hat{X}_1 + p e_2 X_2 \hat{X}_2 + q X_3 (e_3 - 1/m_3) \hat{X}_3 + (q^2 \bar{S}_{33}/m_3) \hat{q} \\ & = -(p q/m_3) (S_{32} + m_3 X_2/q) \bar{p} - (X_1 T_1 \bar{t}_1 + p X_2 T_2 \bar{t}_2 + q X_3 T_3 \bar{t}_3), \end{aligned} \quad (23)$$

where \bar{S}_{33} is the compensated (pure substitution) change of demand for the nontraded good with respect to its own price and S_{32} is the uncompensated change of demand for the nontraded good with respect to the price of importable good.

(13), (14), (15), (21), (22) and (23) constitute the derived form of our model which allow for the presence of VRS as well as nontraded good. By eliminating \hat{w} and \hat{r} from our system, we have the following reduced form :

$$\begin{bmatrix} -\frac{|\theta_{23}|e_1}{|\theta_{12}|} & \frac{|\theta_{13}|e_2}{|\theta_{12}|} & -e_3 & -1 \\ \left(\Lambda_{L1} - \frac{\delta_L e_1}{|\theta_{12}|}\right) & \left(\Lambda_{L2} + \frac{\delta_L e_2}{|\theta_{12}|}\right) & \Lambda_{L3} & 0 \\ \left(\Lambda_{K1} + \frac{\delta_K e_1}{|\theta_{12}|}\right) & \left(\Lambda_{K2} - \frac{\delta_K e_2}{|\theta_{12}|}\right) & \Lambda_{K3} & 0 \\ e_1 X_1 & p e_2 X_2 & q X_3 \left(e_3 - \frac{1}{m_3}\right) & \frac{q^2 \bar{S}_{33}}{m_3} \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_3 \\ \hat{q} \end{bmatrix}$$

$$\begin{aligned}
 & \left[\begin{aligned} & \frac{|\theta_{23}|T_1}{|\theta_{12}|} \bar{t}_1 - \frac{|\theta_{13}|T_2}{|\theta_{12}|} \bar{t}_2 + T_3 \bar{t}_3 \\ & \sum_{j=1}^3 \lambda_{Lj} T_j \bar{t}_j + \delta_L (T_1 \bar{t}_1 - T_2 \bar{t}_2) / |\theta_{12}| \\ & \sum_{j=1}^3 \lambda_{Kj} T_j \bar{t}_j - \delta_K (T_1 \bar{t}_1 - T_2 \bar{t}_2) / |\theta_{12}| \\ & \sum_{j=1}^3 \lambda_{Kj} T_j \bar{t}_j - \delta_K (T_1 \bar{t}_1 - T_2 \bar{t}_2) / |\theta_{12}| \\ & - X_1 T_1 \bar{t}_1 - p X_2 T_2 \bar{t}_2 - q X_3 T_3 \bar{t}_3 \end{aligned} \right] \\
 & + \left[\begin{aligned} & - \frac{|\theta_{13}|}{|\theta_{12}|} \\ & - \frac{\delta_L}{|\theta_{12}|} \\ & \frac{\delta_K}{|\theta_{12}|} \\ & - \frac{pq}{m_3} \left(S_{32} + \frac{m_3 X_2}{q} \right) \end{aligned} \right] \bar{p} \tag{24}
 \end{aligned}$$

where

$$|\theta_{ij}| = \begin{vmatrix} \theta_{Li} & \theta_{Ki} \\ \theta_{Lj} & \theta_{Kj} \end{vmatrix}, \quad (i, j = 1, 2, 3).$$

3. Stability and price-output response

(24) can be solved to obtain the price-output response as:

$$\begin{aligned}
 \frac{\bar{X}_2}{\bar{p}} = \frac{1}{\Delta} & \left[\left\{ q X_3 \left(e_3 - \frac{1}{m_3} \right) - \frac{q^2 \bar{S}_{33} e_3}{m_3} \right\} \frac{B_1}{|\theta_{12}|} \right. \\
 & + \frac{pq}{m_3} \left(S_{32} + \frac{m_3 X_2}{q} \right) |\lambda_{13}| (1 - e_3) \\
 & + \frac{q^2 \bar{S}_{33} |\theta_{13}| |\lambda_{13}| (1 - e_3)}{m_3 |\theta_{12}|} - e_1 \left\{ \frac{pq}{m_3} \left(S_{32} + \frac{m_3 X_2}{q} \right) \frac{B_3}{|\theta_{12}|} \right. \\
 & \left. \left. + \frac{X_1 B_3}{|\theta_{12}|} + \frac{q^2 \bar{S}_{33} B_3}{m_3 |\theta_{12}|} \right\} \right], \tag{25}
 \end{aligned}$$

where

$$\Delta = \left\{ q X_3 \left(e_3 - \frac{1}{m_3} \right) - \frac{q^2 \bar{S}_{33} e_3}{m_3} \right\} \left(|\lambda_{12}| - \frac{e_1 B_2 + e_2 B_1}{|\theta_{12}|} \right)$$

$$\begin{aligned}
& + e_1 \left(X_1 - \frac{q^2 \bar{S}_{33} |\theta_{23}|}{m_3 |\theta_{12}|} \right) \left(|\Lambda_{23}| + \frac{e_2 B_3}{|\theta_{12}|} \right) \\
& - e_2 \left(p X_2 + \frac{q^2 \bar{S}_{33} |\theta_{13}|}{m_3 |\theta_{12}|} \right) \left(|\Lambda_{13}| - \frac{e_1 B_3}{|\theta_{12}|} \right), \\
& B_j = \delta_L \Lambda_{Kj} + \delta_K \Lambda_{Lj} \quad (j=1, 2, 3),
\end{aligned}$$

and

$$|\Lambda_{ij}| = \begin{vmatrix} \Lambda_{Li} & \Lambda_{Lj} \\ \Lambda_{Ki} & \Lambda_{Kj} \end{vmatrix} \quad (i, j=1, 2, 3).$$

The sign of \hat{X}_2/\hat{p} is not certain because of the ambiguity of both the determinant's value of the coefficient matrix Δ and the bracketed term. First, let us discuss the determinant's value which is closely related to the (local) dynamic stability of the system.

According to the assumption of fixed factor endowments, constant tradeable good prices, the existence of a nontraded good and perfect competition, the dynamic adjustment mechanism of the system should be specified as followed:¹⁾

$$\dot{X}_1 = d_1(1 - a_{L1}w - a_{K1}r) \quad (26)$$

$$\dot{X}_2 = d_2(p - a_{L2}w - a_{K2}r) \quad (27)$$

$$\dot{X}_3 = d_3(q - a_{L3}w - a_{K3}r) \quad (28)$$

$$\dot{w} = d_4(a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 - L) \quad (29)$$

$$\dot{r} = d_5(a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 - K) \quad (30)$$

where “.” denotes differentiation with respect to time and d_i is the positive coefficient measuring the speed of adjustment. In the factor market, we are facing a Walrasian adjustment mechanism with the fixed endowment assumption implying that returns will have to adjust. A marshallian adjustment process is assumed with quantities adjusting as the demand price (i.e., the (exogenously given) traded good price and the nontraded good price) differs

¹⁾ We assume that there exists some class of VRS production functions under which the system will be stable, at least for some set of adjustment speed. For the details of this problem, see Mayer (1974).

VARIABLE RETURNS

from the supply price (i.e., the average cost of producing a given commodity) in the traded and nontraded good markets.

The Jacobian matrix of the system of simultaneous equations (26) to (30) is:

$$J = \begin{bmatrix} d_1 & 0 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 & 0 \\ 0 & 0 & d_3 & 0 & 0 \\ 0 & 0 & 0 & d_4 & 0 \\ 0 & 0 & 0 & 0 & d_5 \end{bmatrix} \begin{bmatrix} -a_{L1} & -a_{K1} & e_1/X_1 \\ -a_{L2} & -a_{K2} & 0 \\ -a_{L3} & -a_{K3} & -m_3 e_1/q \bar{S}_{33} \\ -L\delta_L/w & L\delta_L/r & A_{L1} \\ K\delta_K/w & -K\delta_K/r & A_{K1} \\ 0 & 0 \\ pe_2/X_2 & 0 \\ -m_3 e_2 p/q \bar{S}_{33} & qe_3/X - (m_3 e_3 - 1)/\bar{S}_{33} \\ A_{L2} & A_{L3} \\ A_{K2} & A_{K3} \end{bmatrix} \quad (31)$$

where $A_{ij} = (1 - e_j)a_{ij}$. Note that

$$|J| = \frac{d_1 d_2 d_3 d_4 d_5}{-\bar{S}_{33}} \begin{vmatrix} -a_{L1} & -a_{K1} & 0 & e_1/X_1 & 0 & 0 \\ -a_{L2} & -a_{K2} & 0 & 0 & pe_2/X_2 & 0 \\ -a_{L3} & -a_{K3} & 1 & 0 & 0 & qe_3/X_3 \\ -L\delta_L/w & L\delta_L/r & 0 & A_{L1} & A_{L2} & A_{L3} \\ K\delta_K/w & -K\delta_K/r & 0 & A_{K1} & A_{K2} & A_{K3} \\ 0 & 0 & \bar{S}_{33} & m_3 e_1/q & m_3 e_2 p/q & (m_3 e_3 - 1) \end{vmatrix} \quad (32)$$

Therefore, we can show that

$$|J| = -\alpha|\mathcal{Q}| = \alpha|\theta_{12}|\Delta, \quad (33)$$

where $\alpha = d_1 d_2 d_3 d_4 d_5 p L K m_3 / w r q X_1 X_2 X_3 (-\bar{S}_{33}) > 0$ and

$$\mathcal{Q} = \begin{bmatrix} \theta_{L1} & \theta_{K1} & 0 & -e_1 & 0 & 0 \\ \theta_{L2} & \theta_{K2} & 0 & 0 & -e_2 & 0 \\ \theta_{L3} & \theta_{K3} & -1 & 0 & 0 & -e_3 \\ -\delta_L & \delta_L & 0 & \Lambda_{L1} & \Lambda_{L2} & \Lambda_{L3} \\ \delta_K & -\delta_K & 0 & \Lambda_{K1} & \Lambda_{K2} & \Lambda_{K3} \\ 0 & 0 & q^2 \bar{S}_{33} / m_3 & e_1 X_1 & p e_2 X_2 & q X_3 (e_3 - 1 / m_3) \end{bmatrix}$$

According to the Routh-Hurwitz theorem a necessary condition for local stability of the system is $|J|$ is negative. So we assume that our equilibrium is stable which implies that $|J|$ is negative, therefore, $|\theta_{12}|\Delta < 0$ from (33).

Let us return to the price-output response. (25) can be rewritten as:

$$\begin{aligned} \frac{\bar{X}_2}{\bar{p}} = & \frac{1}{|\theta_{12}|\Delta} \left[\frac{pq(1-e_3)|\lambda_{13}||\theta_{12}|}{m_3} \left(S_{32} + \frac{m_3 X_2}{q} \right) \right. \\ & + \frac{q^2(1-e_3)|\lambda_{13}||\theta_{13}|}{m_3} \bar{S}_{33} \\ & \left. - \frac{q X_3}{m_3} \{ (Q_{33} e_3 + 1) B_1 + e_1 B_3 (Q_{32} + Q_{33} + \eta_3) \} \right], \end{aligned} \quad (34)$$

where $Q_{33} = (q/D_3)(\partial D_3/\partial q)$, $Q_{32} = (p/D_3)(\partial D_3/\partial p)$ and $\eta_3 = (Y/D_3)(\partial D_3/\partial Y)$.

(34) shows that the price-output response is not generally normal even in a stable system. Note that the presence of VRS in the importable good sector itself does not affect the result qualitatively. We can mention the following set of sufficient conditions:

- (i) $S_{32} > 0$ (or $(S_{32} + m_3 X_2 / q) < 0$, respectively),
- (ii) $|\theta_{12}||\lambda_{13}| < 0$ (> 0), and
- (iii) $(Q_{33} e_3 + 1) B_1 + e_1 B_3 (Q_{32} + Q_{33} + \eta_3) > 0$ (< 0).

That is, the price-output response will be normal if (i) the importable good is a substitute (or a sufficiently close complement, respectively) for the nontraded good, (ii) the factor intensities of these two good sectors bound (or exceed or fall short of) that of the exportable good sector, and (iii) the certain condition

between the demand for the nontraded good and VRS is satisfied. The former two conditions are familiar in the standard model with nontraded good and the last one is inherent to the presence of VRS which is not easy to interpret unfortunately. In the special cases where $e_1=0$ and $e_3=0$, condition (iii) is reduced to $e_3 \geq -1/Q_{33}$ and $-B_1 \geq e_1 b_3 (Q_{32} + Q_{33} + \eta_3)$, respectively, where $b_3 = \delta_L \lambda_{K3} + \delta_H \lambda_{L3}$.

4. Technical progress and output response

The welfare implications of growth (arising from factor accumulation or technical progress) for an open economy in the presence of VRS has been discussed by Choi and Yu (1985). They also analysed the implications of three types of technical progress on outputs in a VRS framework in Choi and Yu (1987). So, in this section, using the model deployed in the previous sections, we extend the results on technical progress under VRS by analysing the output responses due to technical progress in the nontraded good sector.

We can use (24) to get the following output response due to the Hicks-neutral technical progress in the nontraded good sector :

$$\begin{aligned} \frac{1}{T_3} \frac{\tilde{X}_2}{\tilde{t}_3} &= \frac{1}{|\theta_{12}| \Delta} \left[\left\{ qX_3 \left(e_3 - \frac{1}{m_3} \right) - \frac{q^2 \bar{S}_{33} e_3}{m_3} \right\} \left(\frac{|\Lambda_{13}| |\theta_{12}|}{1 - e_3} - e_1 b_3 \right) \right. \\ &\quad \left. + \left(qX_3 - \frac{q^2 \bar{S}_{33}}{m_3} \right) (|\Lambda_{13}| |\theta_{12}| - e_1 B_3) \right] \\ &= \frac{-qX_3 (Q_{33} + 1) (|\Lambda_{13}| |\theta_{12}| - e_1 B_3)}{m_3 |\theta_{12}| \Delta (1 - e_3)}. \end{aligned} \quad (35)$$

Therefore, the effect of the technical progress in the nontraded good sector on the output of the importable good are summarized as :

	$ \Lambda_{13} \theta_{12} - e_1 B_3 > 0$	$ \Lambda_{13} \theta_{12} - e_1 B_3 < 0$
$-Q_{33} > 1$ (elastic)	-	+
$-Q_{33} < 1$ (inelastic)	+	-

Table 1. The sign of \tilde{X}_2/\tilde{t}_3

Komiya (1967, pp.142-143) demonstrated that the domestic output of the importable increases if the demand for the nontraded good is price-elastic (price-inelastic, respectively) and the importable and nontraded goods have opposite (similar) factor intensities relative to the exportable good. Our result extends Komiya's proposition by allowing for VRS including constant returns to scale (CRS) as a special case. It is interesting that the presence of VRS in the importable and nontraded good sectors themselves does not affect the result qualitatively. The latter part of the condition in Komiya's proposition must be modified due to the presence of VRS in the *exportable* good sector. Suppose that the demand for the nontraded good is price-elastic, i.e., $-Q_{33} > 1$. (1) If CRS prevails in the exportable good sector (i.e., $e_1=0$), the output of the importable increases if the importable and nontraded goods have opposite factor intensities relative to the exportable goods (i.e., $|\Lambda_{13}| |\theta_{12}| < 0$). (2) If increasing returns to scale (IRS) prevail (i.e., $e_1 > 0$), the possibility of increasing the output of the importable *increases* so as to include the case where both goods have *similar* factor intensities relative to the exportable good (the area shaded by horizontal lines in Figure 1). It is obvious that the larger the degree of IRS, the larger the paradoxical possibility. (3) If decreasing returns to scale (DRS) prevail (i.e., $e_1 < 0$), it becomes possible that the output of the importable *decreases* even if both goods have opposite factor intensities relative to the exportable good (the area shaded by dots in Figure 1). So far we have discussed the change in the output of the importable. A similar argument can be provided for the change in the output of the exportable.

Finally, let us investigate the effect of technical progress in the nontraded good sector on its own output. From (24) we have :

$$\frac{1}{T_3} \frac{\hat{X}_3}{\bar{I}_3} = \frac{A}{-|\theta_{12}|A} = \frac{A}{B + (1 - e_3)A}, \quad (36)$$

where

$$A = (qX_3 - q^2 \bar{S}_{33}/m_3)(|\Lambda_{12}| |\theta_{12}| - (e_1 B_2 + e_2 B_1)) \\ + (pX_2 + q^2 \bar{S}_{33} |\theta_{13}| / m_3 |\theta_{12}|)(|\Lambda_{13}| |\theta_{12}| - e_1 B_3) e_2 / (1 - e_3)$$

VARIABLE RETURNS

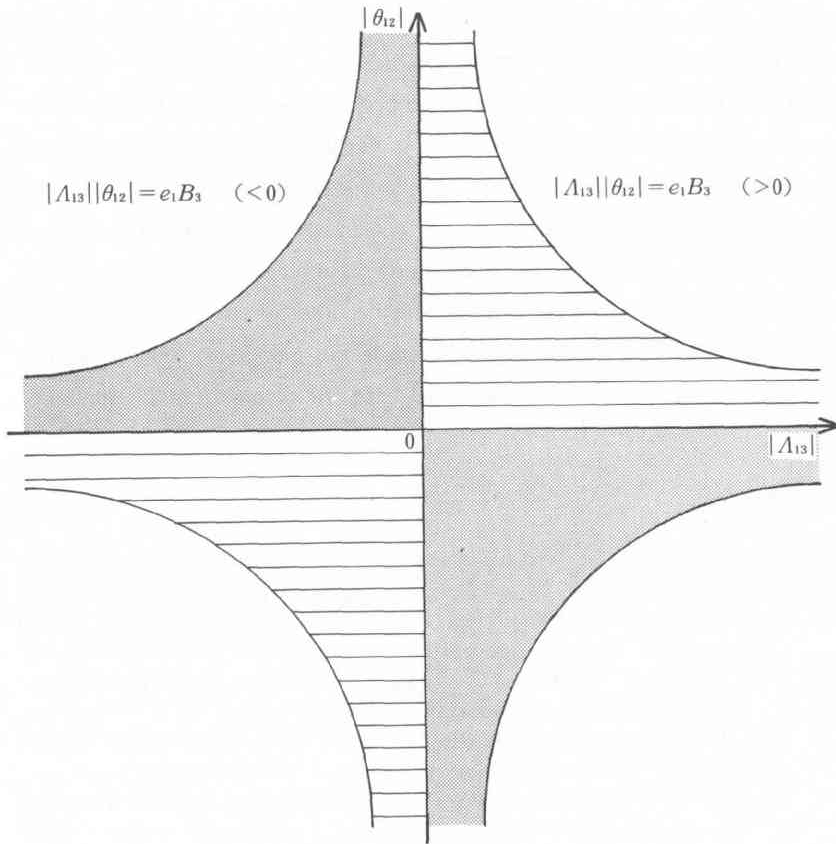


Figure 1.

$$-(X_1 - q^2 \bar{S}_{33} |\theta_{23}| / m_3 |\theta_{12}|) (|\Lambda_{23}| |\theta_{12}| + e_2 B_3) e_1 / (1 - e_3)$$

and

$$B = qX_3(Q_{33} + 1) (|\Lambda_{12}| |\theta_{12}| - (e_1 B_2 + e_2 B_1)) / m_3.$$

The change in the output of the nontraded good depends on the sign of A. However, it is not easy to derive sufficient conditions from A. Noting that we can rewrite the denominator as shown in the latter part of (36), it becomes clear from stability discussed in the previous section that A is positive if B is negative since $-|\theta_{12}|/\Delta > 0$. Therefore, we can say that technical progress in the nontraded good sector increases its own output (1) if both of the traded good sectors have either DRS or CRS the demand for the nontraded good is price-elastic or (2) if at least one of the tradeable good sectors has strong IRS enough such that $|\Lambda_{12}| |\theta_{12}| < (e_1 B_2 + e_2 B_1)$ and the demand for the nontraded good is price-inelastic.

References

1. Batra, Raveendra N., "Protection and Real Wage under Conditions of Variable Returns to Scale." *Oxford Economic Papers* 20, November 1968, 353-360.
2. Batra, Raveendra N., "Nontraded Goods, Factor Market Distortions, and the Gains from Trade." *American Economic Review* 63, March 1973, 706-713.
3. Choi, Jai-Young, "Nontraded Goods, Variable Returns to Scale and Welfare." *Southern Economic Journal* 53, April 1987, 874-883.
4. Choi, Jai-Young and Eden S.H. Yu, "Gains from Trade under Variable Returns to Scale." *Southern Economic Journal* 50, April 1984, 979-992.
5. Choi, Jai-Young and Eden S.H. Yu, "Technical Progress, Terms of Trade and Welfare under Variable Returns to Scale." *Economica* 52, August 1985, 365-377.
6. Choi, Jai-Young and Eden S.H. Yu, "Technical Progress, and Outputs under Variable Returns to Scale." *Economica* 54, May 1987, 249-253.
7. Eaton, Jonathan and Arvind Panagariya, "Gains from Trade under Variable Returns to Scale, Commodity Taxation, Tariffs and Factor Market Distortions." *Journal of International Economics* 9, November 1979, 481-501.
8. Ethier, Wilfred, "Nontrade Goods and the Heckscher-Ohlin Model." *International Economic Review* 13, February 1972, 132-147.
9. Flam, Harry, "The Rybczynski Theorem in a Model with Nontraded goods and Indecomposable Interindustry Flows." *International Economic Review* 20,

VARIABLE RETURNS

October 1979, 661-670.

10. Hazari, B.R., Sgro, P.M. and D. C. Sue, *Notraded and Intermediate Goods and the Pure Theory of International Trade*. London : Croom Helm, 1981.
11. Herberg, Horst and Murray C. Kemp, "Some Implications of Variable Returns to Scale." *Canadian Journal of Economics* 3, August 1969, 403-415.
12. Jones, Ronald W., "Variable Returns to Scale in General Equilibrium Theory." *International Economic Review* 9, October 1968, 261-272.
13. Kakimoto, Sumio and Shigemi Yabuuchi, "The Rybczynski Theorem in a Model with Nontraded Goods and Indecomposable Interindustry Flows: Revisited." *Journal of International Economics* 20, February 1986, 157-169.
14. Kemp, Murray C. *The Pure Theory of International Trade and Investment*. Englewood Cliffs, N.J. : Prentice Hall, 1969.
15. Kemp, Murray C. and Takashi Negishi, "Variable Returns to Scale, Commodity Taxes, Factor Market Distortions and Their Impolcations for Trade Gains." *Swedish Journal of Economics* 72, January 1970, 1-11.
16. Komiya, Ryutaro, "Non-Traded Goods and the Pure Theory of International Trade." *International Economic Review* 8, June 1967, 132-152.
17. Mayer, Wolfgang, "Variable Returns to Scale in General Equilibrium Theory : A Comment." *International Economic Review* 15, February 1974, 225-235.
18. Neary, Peter J., "Dynamic Stability and the Theory of Factor-Market Distortions." *American Economic Review* 68, September 1978, 671-682.
19. Panagariya, Arvind, "Variable Returns to Scale in General Equilibrium Theory Once Again." *Journal of International Economics* 10, November 1980, 499-526.
20. Panagariya, Arvind, "Variable Returns to Scale in Production and Patterns of Specialization." *American Economic Review* 71, March 1981, 221-230.
21. Ray, Alok, "Traded and Non-traded Intermediate Inputs and Rybczynski Theorem." *International Economic Review* 13, October 1972, 523-530.